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Topological Geometry Interpretation of Supersonic Inlet Start/Unstart Based on Catastrophe Theory

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I. Introduction

UNSTART phenomenon is one of the most important issues of hypersonic inlets. If an inlet is not started, the mass capture will be greatly reduced, and the spillage drag will be excessive. For hypersonic air-breathing engines, inlet unstart will cause a large drop of both engine thrust and specific impulse, and it may cause catastrophic damage during hypersonic flight. Over the past decades, many investigations have been conducted to examine the mechanisms of inlet start and unstart. Mayer and Paynter [1,2] simulated an axisymmetric inlet unstart due to the variation of freestream variables such as temperature, velocity, and pressure. Neaves and McRae [3] simulated the 3-D inlet unstart caused by a combustor perturbation. Zha et al. [4,5] investigated unstart transient mechanism of a typical axisymmetric inlet at angle of attack. Cox et al. [6] presented several mechanisms of hypersonic inlet unstart, including backpressure unstart, overcontraction unstart, and angle of attack unstart. Van Wie and Kwok [7], Yuan and Liang [8], and others have observed the nonlinear catastrophe and hysteresis phenomena of inlet start/unstart. These studies have made contributions from different aspects.

As from these studies, the start/unstart process is influenced by many factors, and the physical law is complex with strong nonlinearities such as catastrophe, hysteresis, etc. Because inlet unstart may cause catastrophic damage to the aircraft, it must be avoided and controlled during hypersonic flight. With this consideration, it is important to study the physical law of inlet start/unstart, for example, to study the catastrophe boundaries of inlet start/unstart at different operation conditions (different flight Mach number, angle of attack, flight height, and back pressure). The physical law of inlet start/unstart will be important for further application (i.e., avoid or control inlet unstart).

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As known, the transition between inlet start and unstart is mainly characterized by catastrophe, and such systems can be studied by catastrophe theory. With this consideration, this paper will try to use the topological geometry method in catastrophe theory to study the physical law of inlet start/unstart. In general, catastrophe theory permits the establishment of qualitative and general solutions of the catastrophe behaviors, and it can deal with complex systems with catastrophe properties. In this paper, first, the catastrophe mechanism of inlet start/unstart is analyzed. Second, the nonlinear catastrophe, hysteresis, and bifurcation features of inlet start/unstart are simulated based on the topological geometry method in catastrophe theory, and the catastrophe boundaries of inlet start/unstart are obtained. Third, the physical laws inlet start/unstart are interpreted along typical control routes around the catastrophe boundaries.

II. Catastrophe Mechanism Interpretation of Inlet Start/Unstart

Catastrophe phenomena are the results of dynamic processes that lead to sudden changes. To explore the catastrophe mechanism, we apply the "simplified quasi-one-dimensional" unsteady model of shock motion to qualitatively analyze it. The simplified quasi-one-dimensional unsteady model of shock motion can be described as [9]

$$\dot{x}_s = \lambda x_s + \omega_2^- \theta_2^- + \omega_1^+ \theta_1^+ + \omega_1^- \theta_1^- + \omega_1^e \theta_1^e \tag{1}$$

$$\lambda = -\xi \frac{1}{A} \frac{\mathrm{d}A}{\mathrm{d}x} \tag{2}$$

where x_s is the shock position, θ^+, θ^-, and θ^e are the upstream acoustic wave, downstream acoustic wave, and entropy wave, ω and ξ are the nondimensional coefficients, A is the cross-sectional area, subscripts 1 and 2 denote the locations upstream and downstream of the shock, and λ is the eigenvalue of the equation that determines the stability of the shock [10]. As known, the system will be unstable if λ is positive, and stable if λ is negative. The system will be at a critical state If λ equals to zero. As from the eigenvalue equation, the sign of λ is determined by $\mathrm{d}A/\mathrm{d}x$. Therefore, the stability of the system is mainly determined by the profile of the cross-sectional area. If external disturbances cause the shock to move upstream from the throat, the shock will be unstable and be abruptly expelled ahead of the cowing of the inlet, and there is a catastrophic airflow characteristic in the transition from inlet start to unstart.

III. Simulation of the Physical Laws of Inlet Start/Unstart Based on Catastrophe Theory

A. Numerical Method

The hypersonic inlet model is based on the similar one in [11], and it includes an inlet and a constant area isolator. Figure 1 shows the geometry sketch of inlet model, and the main geometry measurements of the inlet are referred to Table 1.

The computation is performed using the finite-volume technique with upwind discretization to solve the three-dimensional compressible Reynolds-averaged Navier–Stokes equations. The air is considered to be a calorically perfect gas. The space discretization is performed by a cell-centered formulation. A renormalization group k–e turbulence model is implemented for turbulent flows. The near-wall treatment adopting nonequilibrium wall functions is

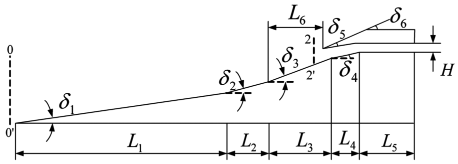


Fig. 1 Geometry sketch of inlet model.

recommended for use in complex flows involving separation, reattachment, and impingement where the mean flow and turbulence are subjected to severe pressure gradients and change rapidly.

To ensure convergence of the numerical solution, the residuals (L_2 norm) are monitored in Fig. 2. The solution can be considered as converged after approximately 3000 iterations. At this stage, the continuity residual, x-velocity residual, and energy residual reach their minimum values after falling for over 4 orders of magnitude. The turbulence residual has a 6 orders of magnitude decrease. An additional convergence criterion enforced in this current analysis requires the difference between computed inflow and outflow mass flux to drop below 0.5%. The evaluation was performed using the coarse mesh.

The performance of a grid sensitivity analysis confirmed that the grid resolution used here is sufficient to capture the physically relevant features. In Fig. 3, the static pressure distributions along the cowl and the ramp surfaces are shown for three different grid-refinement levels: coarse (420×32), medium (754×55) and fine (1358×81); the maximum discrepancy between the three mesh levels is less than 5%. Out of this analysis, the medium grid was selected, and all results shown are computed applying this resolution. To ensure the accuracy of the turbulence flow solution, a value of y^+ below five is realized for the main portion of the wall flow region.

The accuracy of the current numerical investigation is evaluated by comparison with the experimental results. The experimental data is referred to in Fig. 5 of [11]. The surface pressure distributions shown in Fig. 3 allow for a qualitative comparison between numerical and experimental results, where the boundary conditions are $M_0 = 6.4$, $T_0 = 203.5$ K, $p_0 = 3968 P_a$, and $\alpha_0 = 0.75$ deg. Here, a discrepancy in the ramp pressure distributions can be seen, but both overall pressure distributions are consistent. The reason for the discrepancy is probably the deficiency of the turbulence model, the differences between experiment and computation conditions, or the measurements error of sensors. In a word, the computation results of hypersonic inlet accord with the physical conception of the aerodynamics. It can reveal the intersection of oblique shock wave and expansion wave and capture the primary characteristic of internal flowfield.

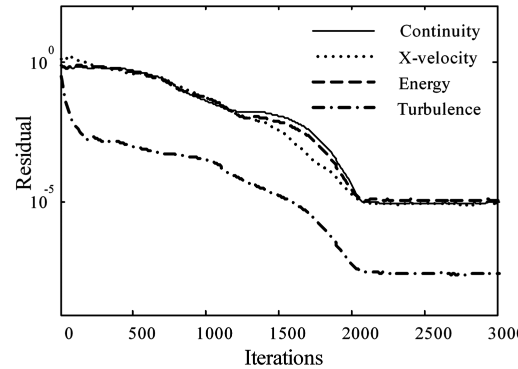


Fig. 2 Residuals for the hypersonic inlet computation.

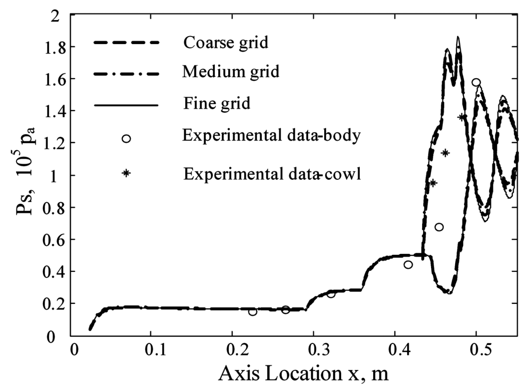


Fig. 3 Surface pressure distributions for refined grid.

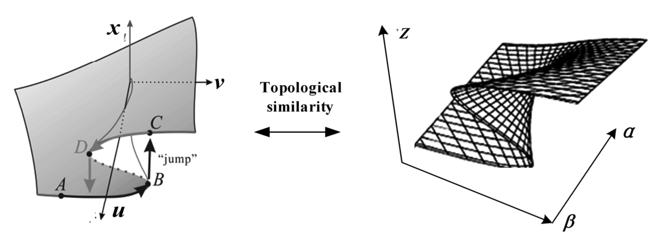


Fig. 4 Topological similarity of the same catastrophe.

B. Catastrophe Theory

In this section, an important concept of the catastrophe theory will be illustrated to investigate the catastrophic phenomenon of inlet start/unstart, and topological geometry method will be introduced to qualitatively explore the physical law of the catastrophic process.

For a better insight into the physical law of inlet start/unstart, the present analysis employs principles of catastrophe theory to probe the relationship between mathematical models and the physical systems of hypersonic inlet. Catastrophe theory, among other advantages, permits the establishment of qualitative and general solutions. The theory is a mathematical tool developed by Thom, a French mathematician who founded catastrophe theory in the 1960s, and it can deal with complex systems with properties of discontinuities directly even without reference to any specific underlying mechanism [12,13]. This attractive property makes it especially appropriate for the interpretation of systems whose inner workings may not be known, as is usually the case in the study of complex systems. Catastrophe theory is built on the base of singularity theory and topology theory, and the essence of catastrophe theory is the topology of singular spaces.

Singularity theory provides a taxonomy of singular objects and a binary key to identify them, and it gives significant proof on the invariance principles of singularity under smooth mapping; hence, it is a classification science. Based on the conception of singularity classification, catastrophe theory proves with topological theory that when the space dimension is no more than four, there exist only seven classes of elementary catastrophes.

The conception of topology here means that if two systems have the same catastrophe type, they will be similar in topological configuration, as may be shown in Fig. 4. This is very important, for example, because if we can determine the catastrophe type of inlet start/unstart, we can get to know its topological configuration. With this help, we can study the concrete catastrophe laws of inlet start/unstart according to the topological configuration.

Catastrophe theory provides a classification of catastrophes and can supply the tools for the comprehension and detection of the catastrophe process. In the following sections, the topological geometry method will be used to analyze the catastrophe laws of inlet start/unstart.

Table 1 Geometry parameters of hypersonic inlet

Length, L	L_1	L_2	L_3	L_4	L_5	L_6
Unit, m	0.267	0.068	0.085	0.035	0.095	0.074
Angle, δ	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6
Unit, deg	10	15	20	14	10.2	24

C. Simulation Results

The physical laws of inlet start/unstart are complex because of nonlinearity. However, the topological geometry method provided by catastrophe theory may help simulate the physical laws of the catastrophe system of inlet start/unstart.

As far as hypersonic inlet is concerned, the transition between inlet start and unstart is mainly controlled by the variation of flight Mach number, angle of attack, flight height, back pressure, etc. In this paper, only two variables of fight Mach number and angle of attack are considered as control variables, and mass flow rate coefficient is selected as the state variable, which can be used to describe the catastrophe characteristic of inlet start/unstart. On this condition, the catastrophe type of inlet start/unstart will be belong to cusp catastrophe, according to catastrophe theory. Accordingly, the topological configuration of inlet start/unstart will be the same as that of the elementary cusp catastrophe. In other words, this means that the two systems have the similar catastrophe configuration.

With this direction, this paper has simulated the concrete catastrophe characteristic of inlet start/unstart as shown in Fig. 5b, which shows the change of mass flow rate coefficient with flight Mach number and angle of attack. The simulation results are obtained by two adverse routes along the axis of angle of attack, that is, first we increase the angle of attack gradually at a certain flight Mach number and obtain a series of simulation results, and second we decrease the angle of attack gradually and obtain another series of simulation results. From the simulation results, we can see that there are both continuous processes (at low flight Mach numbers of 2.0, 2.5, and 2.8) and discontinuous processes (at high flight Mach numbers of 3.0, 3.5, 4.0, 4.5, 5, and 5.5) in the transition between inlet start and unstart. As from the general configuration, the physical system of inlet start/unstart has the similar catastrophe configuration as the elementary cusp catastrophe shown in Fig. 5a.

IV. Physical Laws Interpretation of Inlet Start/Unstart Based on Catastrophe Theory

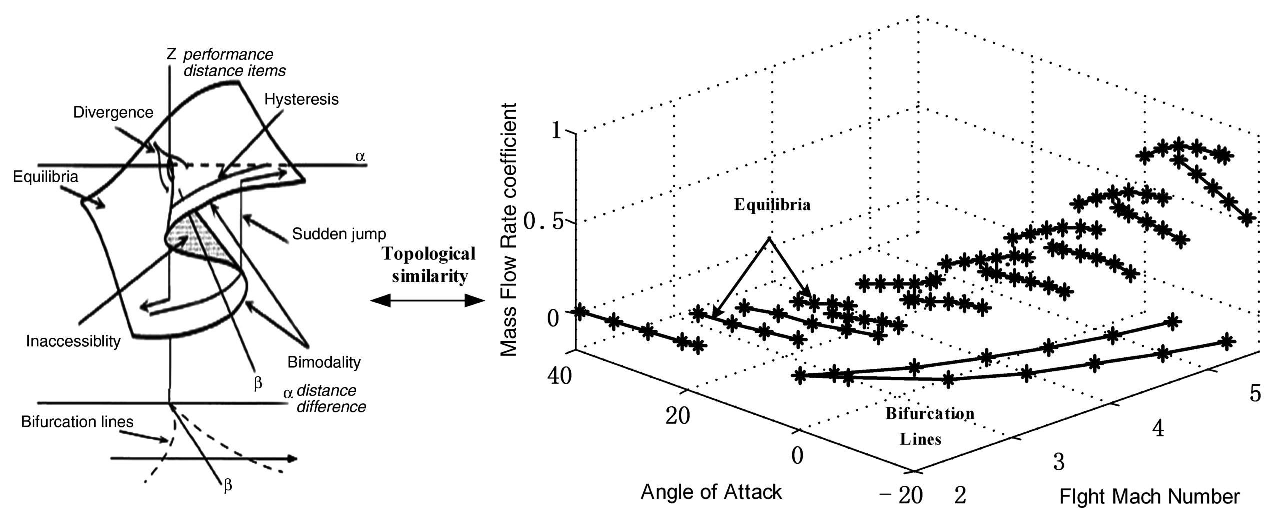
In this section, the catastrophe, hysteresis, and bifurcation laws will be interpreted based on catastrophe theory.

A. Bifurcation (Catastrophe Boundary) Interpretation

In the viewpoint of catastrophe theory, the existence of singular points has invariance principles under topology transformation [12]. Therefore, the catastrophe characteristic of inlet start/unstart can be interpreted based on the topological geometry space of cusp catastrophe.

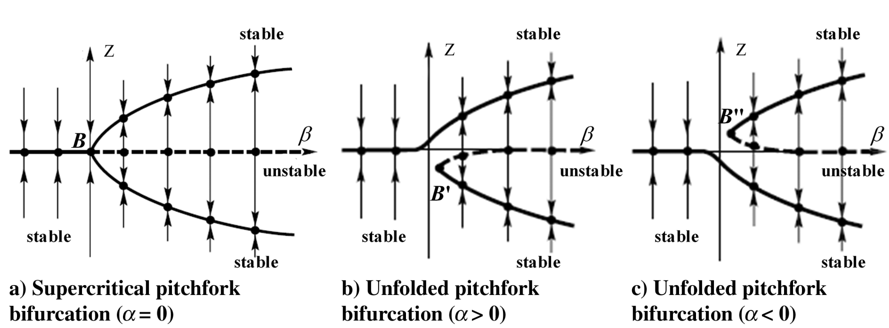
As shown in Fig. 5b, the occurrence of inlet start/unstart transition corresponds to the bifurcation phenomena. That is, when flight Mach number is high (at flight Mach numbers 3.0, 3.5, 4.0, 4.5, 5.0, and 5.5), there exists a two-state catastrophe structure (inlet start and unstart) with the change of angle of attack. However, when flight Mach number is relatively low (at flight Mach numbers 2.0, 2.5, and 2.8), there exists only one state (inlet unstart) with the change of angle of attack, and there is no catastrophe in the process. This is the characteristic of bifurcation [12].

To interpret the bifurcation law of inlet start/unstart, topological geometry method is used. As shown in Fig. 5a of the topological geometry space of cusp catastrophe, when $\alpha = 0$, where α is regarded as the topological transformation function of flight Mach number and angle of attack for the system of inlet start/unstart transition according to the catastrophe theory, the system of inlet start/unstart transition has a supercritical pitchfork bifurcation as shown in Fig. 6a. It can be seen that in this bifurcation that both the upper and lower branch of the pitchfork bifurcation have stable equilibrium points, which correspond to the two stable states: inlet start and unstart as shown in Fig. 5b. But the middle branch is shown



a) Elementary cusp catastrophe b) Catastrophe characteristic of inlet start/unstart

Fig. 5 Topological similarity of the same cusp catastrophe.



a) Supercritical pitchfork bifurcation ($\alpha = 0$) b) Unfolded pitchfork bifurcation ($\alpha > 0$) c) Unfolded pitchfork bifurcation ($\alpha < 0$)

Fig. 6 Pitchfork bifurcation and unfolded pitchfork bifurcation of inlet start/unstart.

dashed to denote that it is unstable, which corresponds to the unstable state when shock moves to the convergent channel before the inlet throat, in which the shock motion is unstable as discussed in Sec. II of this paper. Because the state is unstable, they are regarded as "inaccessible" in the catastrophe theory. Another branch in Fig. 6a, described as $\beta > 0$, has only stable equilibrium points, which corresponds to the state of inlet unstart at low flight Mach number which is described as "low Mach unstart," and no catastrophes occur in this condition. It is interesting to find that when $\alpha \neq 0$, the pitchfork bifurcation unfolds as shown in Figs. 6b and 6c. That is, when $\alpha \neq 0$, there will be a hysteresis along the β direction, which causes a sudden change from one stable branch to the other stable branch, for example, from the lower to the upper branch when β is decreased to the critical point as shown in Fig. 6b. However, there is no hysteresis along the β direction when $\alpha = 0$. This phenomena can be mathematically concluded and easily observed in the system of inlet start/unstart when the topological transformation function α of flight Mach number and angle of attack does not equal to 0. In Fig. 6, the bifurcation points, such as B, B', and B'' form the bifurcation lines. Accordingly, the bifurcation points, such as $\bar{B}$, $\bar{B}'$, and $\bar{B}''$ form the bifurcation lines of inlet start/unstart as shown in Fig. 7. In fact, the bifurcation lines form the catastrophe boundaries of inlet start/unstart at which inlet will transit between inlet start and unstart. The catastrophe boundaries can be used to judge when the transitions occur between inlet start and unstart.

B. Catastrophe and Hysteresis Interpretation

According to the topological geometry space of cusp catastrophe in Fig. 5a, at a fixed splitting factor $\beta > 0$, when we increase the asymmetry factor α, there will be a sudden jump in the depended factor Z; when we change α in the adverse route, there will still be a sudden jump in Z. However, the positions of the two sudden jumps are different, and there is a hysteresis loop in the process.

As shown in Fig. 5b, catastrophe and hysteresis loop exist in the transition between inlet start and unstart. As shown in Fig. 7, the catastrophe boundaries describe the positions at which catastrophe occurs, that is, at the position, the inlet will transit between inlet start and unstart. On the other hand, the existence of bifurcation also means that there will be no catastrophe on some conditions. Therefore, the catastrophe characteristic of inlet start/unstart will change with the control variables. To investigate the catastrophe laws of inlet start/unstart, the typical routes of the control variables are constructed around the catastrophe boundaries of inlet start/unstart to observe the corresponding catastrophe laws.

Figure 8 shows five kinds of typical control routes, and each route has two directions to determine whether hysteresis exists.

Figure 9 shows the change of mass flow rate coefficient along control route 1 at the fixed flight Mach number 2.5. It can be seen that the change of mass flow rate coefficient is continuous, and there is no catastrophe and hysteresis along the control route back and forth. On this condition, the inlet operates in the state of low Mach number unstart.

Figures 10 and 11 show, respectively, the change of mass flow rate coefficient along control routes 2 and 3, where angle of attack and flight Mach number change simultaneously. It can be seen that the change of mass flow rate coefficient is continuous on both routes, and there is no catastrophe and hysteresis along the control routes back and forth. In Fig. 10, the inlet operates in the state of high Mach number start, and the state can be shown in Fig. 5 by the upper cusp surface of the folding section. In Fig. 11, the inlet operates in the state of high Mach number unstart, and the state can be shown in Fig. 5 by the down cusp surface of the folding section.

Figures 12 and 13 show, respectively, the change of mass flow rate coefficient along control routes 4 and 5. On these conditions, the control routes walk through the catastrophe boundaries. It can be seen in Fig. 12 that when the flight Mach number is increased to around 4.4, the system transits from inlet unstart to start, and when the flight Mach number is then decreased to around 3.5, the system transits from inlet start to unstart. There is catastrophe and hysteresis loop in the transition between inlet start and unstart with the change

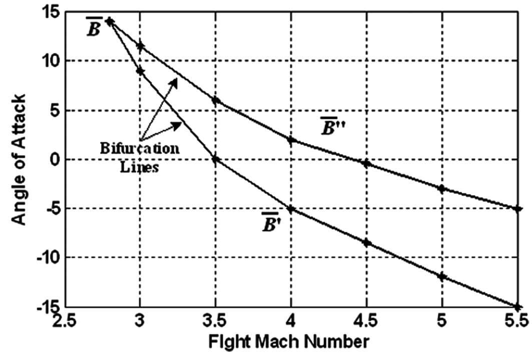


Fig. 7 Catastrophe boundaries of inlet start/unstart.

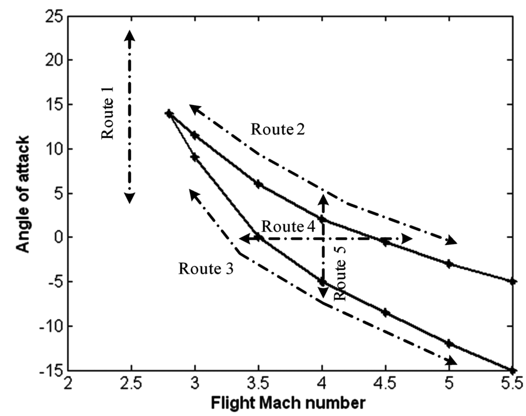


Fig. 8 Different control route around the bifurcation lines of inlet start/unstart.

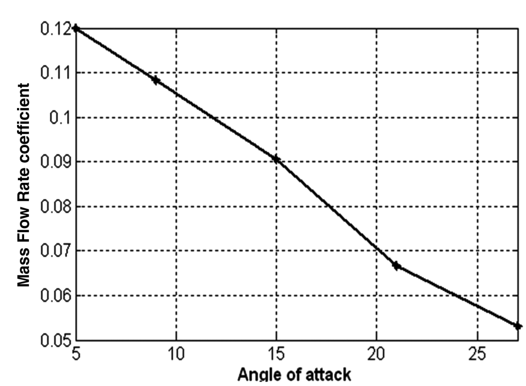


Fig. 9 Change of mass flow rate coefficient along route 1.

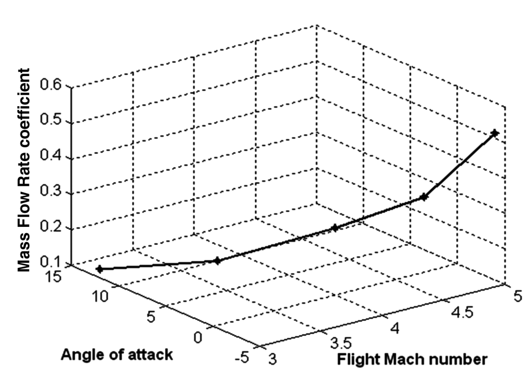


Fig. 10 Change of mass flow rate coefficient along route 2.

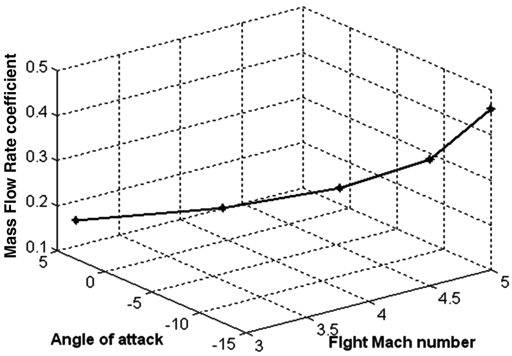


Fig. 11 Change of mass flow rate coefficient along route 3.

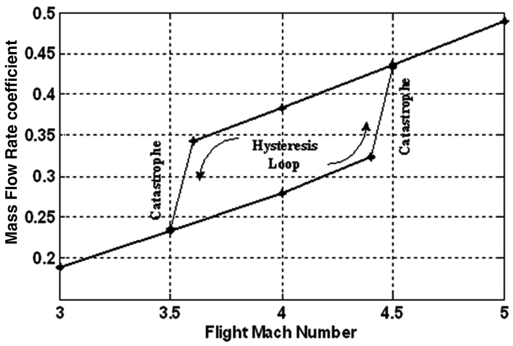


Fig. 12 Change of mass flow rate coefficient along route 4.

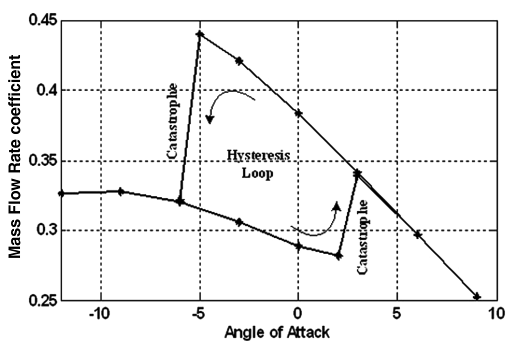


Fig. 13 Change of mass flow rate coefficient along route 5.

of flight Mach number. It can also be seen from Fig. 13 that when the angle of attack is increased to around 2 deg, the system transits from inlet unstart to start, and when the angle of attack is then decreased to around -5 deg, the system transits from inlet start to unstart. There is also catastrophe and hysteresis loop in the transition between inlet start and unstart with the change of angle of attack. In Figs. 12 and 13, the inlet operates in the state of either high Mach number start or high Mach number unstart, and the states can be shown in Fig. 5 by folding section of the cusp surface.

V. Conclusions

Based on catastrophe theory, this paper sets up a topological geometry method to obtain the general catastrophe laws of inlet start/unstart. With the catastrophe laws, the catastrophe boundaries of inlet start/unstart are obtained, and the existent catastrophe, hysteresis, and bifurcation laws are interpreted under typical control routes around the catastrophe boundaries. The interpretation shows that the catastrophe laws of inlet start/unstart are greatly influenced by the control routes. The general catastrophe laws of inlet start/unstart may be useful for further application (i.e., avoid or control inlet unstart).

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